„Ekonomista" 2015, nr 4
http://www.ekonomista.info.pl

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Multifactor-efficiency of the Fama-French Portfolios Formed on the Warsaw Stock Exchange: Bootstrap Method Application

Introduction

The classic CAPM assumes that investors hold Markowitz' mean-variance-efficient (MVE) portfolios. However, the ICAPM investors form multifactor-efficient (ME) portfolio that combines MVE portfolio and hedging portfolios. Despite the fact that Merton (1973) and Long (1974) show that the CAPM is a special case of the ICAPM, not every multifactor pricing model can be interpreted as the ICAPM application. The ICAPM is based on general utility function $U(C_{t-1}, w_t | K_t)$, which depends on: consumptions $C_{t-1} = (..., c_{t-2}, c_{t-1})$ including time $t-1$, wealth at the next point in time w_t and S state variables $K_t = (k_{1t}, k_{2t}, \ldots, k_{St})$, to be observed at t.

The ICAPM investor builds multifactor-minimum-variance (MMV) portfolios defined by percentages of securities minimizing variance of portfolio returns according to the following relation:

$$\min_{X_{ip}} \left[\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ip} x_{jp} \sigma_{ij} \right], \tag{1}$$

under the conditions:

$$\sum_{i=1}^{N} x_{ip} b_{is} = b_{ps}, \; s = 1, ..., S, \tag{1a}$$

$$\sum_{i=1}^{N} x_{ip} E(r_{it}) = E(r_{pt}), \tag{1b}$$

$$\sum_{i=1}^{N} x_{ip} = 1, \tag{1c}$$

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where b_{ps} are the loadings of state variables k_{st} in the following regression:

$$r_{pt} = E(r_{pt}) + \sum_{s=1}^{S} b_{ps} k_{st} + \varepsilon_{pt}, \quad (2)$$

and b_{is} is the component of vector b_{ps} corresponding to security i. $E(r_{pt})$ and $B_p = (b_{p1}, \ldots, b_{pS})'$ are assumed by investor. However, in practice the ICAPM investor chooses greater wealth and maximizes $E(r_{pt})$ for assumed σ_p^2 and B_p, building ME portfolios. Thus, the determined ME portfolios represent a boundary, often called an efficient frontier, of the set of means and variances of returns on all portfolios of given securities.

The basis of ICAPM is a statement that covariance between stock returns and state variables allows investors to choose a portfolio that will hedge uncertainty of future investments. Thus, the main assumption of pricing in light of the ICAPM is that hedging portfolios are built on the basis of state variables forecasting for all future states of economy. State variables should take into account and hedge various decisions of all investors, and they are defined at the beginning of the investment (see: Fama, 1996). That is why ICAPM factors should be defined using state variables. The scientific literature does not specify the precise relationship between model factors and state variables. However, there are some indications about that in the fundamental studies of Fama (1970) or Merton (1973). Also, Campbell (1996) points out that ICAPM factors should be related to innovations in state variables, forecasting future investment opportunities. Closer ties of ICAPM factors and state variables are indicated by Maio and Santa-Clara (2012). However, the assumed strong restrictions for factors, without detailed justification, reject six of the eight tested models as the ICAPM applications. According to Maio and Santa-Clara works, only the three-factor Fama and French (FF) and Carhart (1997) models, tested on the American market, can be justified with the ICAPM.

In the light of the above consideration, it can be concluded that forming of ME portfolio is a necessary, but not sufficient, condition of stock pricing in light of the ICAPM. Testing the stock pricing that could be observed in the conditions of ICAPM validity should be referred not only to an analysis of ME of a given portfolio. Predictability tests of expected returns by given state variables are also necessary. Moreover, systematic risk components of the formed portfolios should be priced in the pooled time series and cross-section estimation of the respective multifactor model.

Attention should be paid to the fact that ME of a given portfolio as well as pricing of systematic risk are usually tested using ex post data. These boundary conditions and lack of knowledge of the exact composition of the market portfolio prevent the running of accurate tests of ICAPM.

ME can be tested using the asymptotic χ^2 distribution corresponding to the Wald statistics. However, for finite samples, the Wald test tends to over-reject the ME portfolio hypothesis (Chou and Zhou, 2006, p. 221). To correct this, Gibbons et al. (1989) (GRS) show an exact test that is valid theoretically only under the normality assumption and can be applied to a small sample. Affleck-Graves and McDonald (1989) find that when normality is strongly rejected the power of the GRS test can be seriously impaired. The asymptotic Wald test can be applied only for large samples under iid assumption. The stock pricing applications in emerging markets are tested using samples of a moderate size for which only iid conditions can be assumed but normality is usually rejected. The true distribution of the returns is never known, therefore there is a need to consider good approximations. The bootstrap method can overcome this problem.

While working on statistical inference for time series, one frequently encounters serious problems with traditional confidence intervals based on asymptotic results. It is now a common knowledge among time series specialists that bootstrap methods provide the most powerful tools for confidence intervals constructions. The advantage of bootstrap methods over other approaches, including a Bayesian one, is clear. Under very general model assumptions, without specifying a parametric model, we “let the data speak for themselves” while constructing confidence intervals. Such advantage of boostrap methods for time series inference has been known for at least ten years. We direct the readers to the famous monographs of Politis (1999) or Lahiri (2003) for a detailed technical account of the above issue.

The essence of the bootstrap method is its ability to approximate the sampling distribution of the test statistics using the data from the sample. This provides a better approximation than the classical central limit theorem and the normal distribution. Chou and Zhou (2006) show various advantages of bootstrap approximations as compared to the classical normal distribution.

In this work we test the three-factor model proposed by Fama and French (1993) (FF). We expect that for stocks listed on the Warsaw Stock Exchange (WSE) the following conjectures are true:

Conjecture 1
The three-factor Fama and French (1993) model generates ME portfolios.

Conjecture 2
Systematic risk components of Fama and French portfolios are priced.
The evidence of Conjectures 1 and 2 leads to Conjecture 3.

Conjecture 3
The valuation of Fama and French portfolios is consistent with the pricing that could be observed in the conditions of ICAPM validity.

The study of the above conjectures is the main objective of this paper. Also, the investigated procedure allows us to asses distributions of risk components for tested portfolios and components of risk premium. This allows us to get a lot of useful information for investors and is an additional aim of the study.

In the paper we use the bootstrap method to test the FF application. Bootstrap and other resampling methods are extensively studied also in time series context (Leśkow et al, 2008).

Section 1 discusses theoretical methods for testing multifactor-efficiency of a given portfolio. Section 2 proposes the possible use of the bootstrap method in finance. Section 3 presents a procedure for juxtaposing data, and results of calculations. The final section presents conclusions.

1. Multifactor-efficiency restrictions

Multifactor application of ICAPM can be described by the regressions (1) and (2) of the following two-step procedure:

$$r_{it} = \alpha_i + \beta_i f_t + e_{it}, \quad \forall i = 1, \dots, N; \quad t = 1, \dots, T, \tag{1}$$

$$r_{it} = \gamma_0 + \gamma_1 \widehat{\beta}_i + \varepsilon_{it}, \quad i = 1, \dots, N; \quad t = 1, \dots, T, \tag{2}$$

where r_{it} is the excess over the risk-free rate on asset i in period t, f_t is the k-vector of factors, β_i is the k-vector of first pass regression parameters for asset i, γ_i is the k-vector of the second pass regression parameters, and e_{it} and ε_{it} are error components. Here, N is the number of assets, and T is the number of observations.

ME of the portfolio implies the following equation:

$$E(R_t) = \beta E(f_t), \tag{3}$$

where R_t is N-vector of the excess returns and $\beta = (\beta_1, \ldots, \beta_N)'$.

The pricing restriction (3) can be formulated as the hypothesis testing problem:

$$H_0\text{: } \alpha = 0, \text{ where } \alpha = (\alpha_1, \ldots, \alpha_N)'.$$

Such a null hypothesis can be tested using the asymptotic χ^2 distribution corresponding to the following Wald statistic:

$$W = \hat{\alpha}' var\left[\hat{\alpha}\right]^{-1} \hat{\alpha}, \quad \sim \chi^2_N. \tag{4}$$

If the errors *eit* defined in (1) are iid then (4) is of the form (Cochrane, 2001, pp. 217–219):

$$W = \frac{T}{1 + E(f_t)' var[f_t]^{-1} E(f_t)} \hat{a}' \hat{\Sigma}_e^{-1} \hat{\alpha}, \quad \sim \chi^2_N, \tag{5}$$

where $\hat{\Sigma}_e = \hat{e}'\hat{e}/(T - k - 1)$, and $\hat{e}$ is the $T \times N$ matrix of residuals.

In practice, applying the Wald test or GRS method requires estimating the matrix Σ_e. This, in turn, induces imposing the normality assumption on the random error terms in Eqs. (1) and (2) to ensure that the statistic $\hat{t} = \hat{\theta}_i / se\left(\hat{\theta}_i\right)$ has a t-Student distribution.[1] In reality, however, the exact distribution of $\hat{t}$ is not known. The bootstrap method can overcome this problem.

2. Bootstrap method approach

To overcome the above mentioned problems with normality assumption or the asymptotic approach we propose the bootstrap method, described by Efron and Tibshirani (1993). The bootstrap method, through resampling algorithm, is able to approximate the finite sample distribution of the parameter estimates without the normality assumption. Moreover, as evidenced in Efron and Tibshirani (1993), for finite samples it provides more reliable results than the normal approximation.

For the convenience of the reader, one should address the problem of bootstrap and other resampling techniques for time series. First of all, the classical bootstrap as applied directly to time series does not work, since it does not recover the dependence structure of the time series data. In view of that, there are usually two general approaches: to use blocking techniques (for example moving block bootstrap (see e.g. Dehay, Dudek and Leskow (2014)) or to represent time series via some structural equations with independent errors, like autoregressive models (see Lahiri (2003)). In our paper, we follow the second approach and, therefore, we take advantage of the simplicity of the simple nonparametric bootstrap in the time series approach. The validity of such approach is well known (see again the monograph of Lahiri (2003).

[1] se $(\hat{\theta}_i)$ is the standard error of $\hat{\theta}_i$.

For regression-type models, three bootstrap-type algorithms can be used:

Case 1: the model errors are iid and the factors are treated as fixed constants. In this case, the fitted residuals are resampled.[2]

Case 2: the assets returns and the factors are jointly iid. Then, the returns are resampled.

Case 3: the factors and the model errors are iid. The factors and the fitted residuals are resampled.

Hall (1992) explains that bootstrap approximations provide more accurate distributions for the first and second case.

The bootstrap test, in the case of the first assumption, can be designed as follows:

1. Estimate the parameters of regressions (1) and (2) by a chosen asymptotic method. In the bootstrap procedure, we call these regressions: „null” regressions. Under such null regression
   a) determine the model residuals $\hat{e}_{it}$;
   b) calculate the Wald statistic:

$$W = \hat{\alpha}' var[\hat{\alpha}]^{-1} \hat{\alpha}. \tag{6}$$

2. Repeat the following procedure large number of times.
   a) draw the residuals e^*_{it}, $t = 1, \ldots, T$ from $\hat{e}_{it}$ with replacement;
   b) generate the bootstrap returns as follows:

$$r^*_{it} = \alpha_i + \beta_i f_t + e^*_{it}. \tag{7}$$

   c) estimate the bootstrap parameters of the first path of the model, α^*_i and β^*_i, of the following regression:

$$r^*_{it} = \alpha^*_i + \beta^*_i f_t + e_{it}; \tag{8}$$

   d) estimate the bootstrap parameters, of the second path of the model, γ^*_0 and γ^*_1, of the following regression:

$$r^*_{it} = \gamma^*_0 + \gamma^*_1 \hat{\beta}^*_i + \varepsilon_{it}; \tag{9}$$

   e) calculate the bootstrapped Wald statistic:

$$W^* = \frac{T}{1 + E(f_t)' var[f_t]^{-1} E(f_t)} (\hat{\alpha}^*)' \hat{\Sigma}_e^{-1} \hat{\alpha}^*. \tag{10}$$

   f) calculate the percentages of α^*_i's, β^*_i's, γ^*_0's, γ^*_1's, and W^*'s that are greater than α_i and β_i, γ_0, γ_1, and W, respectively. The percentages are the p-values of the bootstrap test.

The successful application of resampling algorithms such as bootstrap requires proving their consistency, that is proving that quantiles obtained from computer-generated resampling algorithms correspond to normal quantiles for large samples. The consistency of the above mentioned methods is based on consistency of nonparametric bootstrap method presented e.g. in Davison and Hinkley (1999).

3. Data and experimental results

In this section, we test the FF model on the basis of the Warsaw Stock Exchange (WSE) in 1995–2010. Research studies testing the Capital Asset Pricing Model in the Polish market has been conducted by Jajuga (2000), Bołt and Miłobędzki (2002), Wolski (2004),

[2] The errors are not observable, thus fitted residuals are used.

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Grotowski (2004), Zarzecki et al. (2004–2005), Urbański (2011) and Waszczuk (2013) among others. Bołt and Miłobędzki (2002) verify the hypothesis of intercept insignificance of statistical model, testing the classic CAPM using monthly returns in 1995–1999. Using GRS test, the authors show that the CAPM generates MVE portfolios. Wolski (2004) and Grotowski (2004) test the CAPM using Fama and MacBeth (1973) method. However, the results of their study do not reject a hypothesis about zero value of the risk price. Urbański (2011, 2012) proposes the aggregated two and three-factor model as the ICAPM applications. The performed tests confirm the hypothesis about positive value of risk price components in 1996–2010. Waszczuk (2013) investigates distributions of stock returns related to chosen stock fundamentals, momentum and liquidity in 2002–2011. Additionally, the author assesses returns and factor loadings by classic CAPM and FF model, running OLS regression and ignoring the possible impacts of autocorrelation and hetroscedascity. In the paper of Waszczuk values of FF factor loadings are documented only for portfolios formed on momentum. They are significantly different from zero for all five portfolios for market factor, and for two portfolios for *HML* and *SMB*. Waszczuk (2013) also states that Polish domestic *SMB* and *HML* are not correlated with their U.S. equivalents, which in the light of Maio and Santa-Clara (2012) work is an additional justification for our studies.

A rapid increase in the number of WSE companies is recorded after 2004, following Poland's accession to the EU. However, it has been accompanied by an increase in the number of speculative stocks whose returns are not linked to their financial results. Consequently, the tests are performed for two modes. The mode 1 considers all WSE stocks except of companies characterized by a negative book value. In the mode 2, we eliminate speculative stocks meeting one of the following boundary conditions: (a) $MV/BV > 100$, (b) $ROE < 0$ and $BV > 0$ and $MV/BV > 30$ and $r_{it} > 0$, where *MV* is the stock market value, *ROE* is the return on book value (*BV*).[3] The speculative stocks appear from Q1 of 2005. The number of analyzed companies decreased from 10% in 2005 to 30% in 2010, after exclusion of speculative stocks. All stock returns are calculated in excess of 91 – day Polish Treasury bill return (*RF*). We test FF three factor model (see: Fama and French (1993)) in which the three common risk factors ($RM_t - RF_t, f^{HML}$ and f^{SMB})[4] are used to explain the changes of average returns of the chosen portfolios. The market return (*RM*) is evaluated by the return on the WIG/ WSE index.

The model parameters are determined for full-sample observations and for two separate sub-periods: 1995–2005, the years preceding Poland's accession to the EU, and 2005–2010, the years of Poland's membership in the UE. Data referring to the fundamental results of the inspected companies is taken from the database drawn up by Notoria Serwis Sp. z o.o. Data for defining returns on securities is provided by the Warsaw Stock Exchange.

The data presented by Urbański (2012) indicate that the WSE is among the average-sized European stock exchanges. It is justifies the choice of the WSE as an area for analyzing the returns on Central Europe's emerging markets.

[3] The values 100 and 30 are assumed arbitrarily. If $MV/BV = 100$, a stock has to be extremely speculative. If $MV/BV = 30$ and *rit* assumes positive values, a stock with a huge market value or a very small book value seems to be speculative too.

[4] f_t^{HML} (high minus low) is the difference between the simple average of the returns on the two high-*BV/MV* portfolios ($(BV/MV)_{5t}$ and $(BV/MV)_{4t}$) and the average of the returns on the two low-*BV/MV* portfolios ($(BV/MV)_{1t}$ and $(BV/MV)_{2t}$). f_t^{SMB} (small minus big) is the difference between the simple average of the returns on the two small-*CAP* portfolios (CAP_{5t} and CAP_{4t}) and the average of the returns on the two big-*CAP* portfolios (CAP_{1t} and CAP_{2t}).

The entire sample comprises 56 quarterly investment periods from 10 May 1996 to 12 May 2010. The first sub-period covers 36 quarters from 10 May 1996 to 19 May 2005. The second sub-period covers 20 quarters from 19 May 2005 to 15 May 2010. The inspected securities are divided into quintile portfolios built on the basis of *BV*/*MV* and capitalization (*CAP*).[5] *BV*/*MV* and *CAP* are calculated for all analyzed securities at the beginning of each investment period in which the return is to be calculated. *BV*/*MV* and *CAP* for portfolios constitute average arithmetical values of these functions of various securities of the portfolio. Returns on given portfolios are average stock returns weighted by market capitalizations. The factors f_t are assigned to company portfolios.

The bootstrap quantile is based on 10 000 resamples of the data.

Absolute values of correlation coefficient between the response variable and explanatory variables range from 0.15 to 0.92.

Absolute values of the correlation coefficient between explanatory variables of FF model equal to 0.46 for full-sample observations and 0.55 for sub-period 1995–2005, and 0.44 for sub-period 2005–2010.[6] For the second sub-period the correlation between $RM_t - RF_t$ and f_t^{HML} equals 0.44, and between $RM_t - RF_t$ and f_t^{SMB} – 0.20.[7] It is possible, therefore, to duplicate information. The orthogonalized market factors are defined using the following regression:

$$RM_t - RF_t = \alpha + \beta_{HML} f_t^{HML} + \beta_{SMB} f_t^{SMB} + e_t; \quad t = 1, ..., T, \tag{11}$$

where:

Mode = 1; full-sample
$\alpha = 0.00$, (93.38%) $\beta_{HML} = -0.31$, (5.84%) $\beta_{SMB} = 0.28$, (11.90%) $R^2 = 7.58\%$;

Mode=2; full-sample
$\alpha = 0.00$, (99.33%) $\beta_{HML} = -0.26$, (10.71%) $\beta_{SMB} = 0.25$, (17.49%) $R^2 = 5.75$;

Mode 1; first sub-period
$\alpha = -0.01$, (77.85%) $\beta_{HML} = -0.20$, (28.33%) $\beta_{SMB} = 0.07$, (75.05%) $R^2=3.78\%$;

Mode 1; second sub-period
$\alpha = 0.00$, (92.18%) $\beta_{HML} = -0.43$, (24.59%) $\beta_{SMB} = 0.71$, (4.31%) $R^2=24.89\%$;

Mode 2; second sub-period
$\alpha = -0.01$, (76.86%) $\beta_{HML} = -0.11$, (76.83%) $\beta_{SMB} = 0.73$, (6.06%) $R^2=19.84\%$.

Under the regression model (11) the values of variable loadings are included for all tested periods. The corresponding *p*-values appear in brackets. Regression (11), especially for the second sub-period, contains a higher explanatory power. The value of the orthogonalized market factor is defined as follows: [8]

[5] The tested securities are divided into quintile portfolios in one direction. 5 portfolios are formed on *BV*/*MV* and 5 on *CAP*. The capitalization is a product of the stock market value and the company stock number.

[6] Corresponding *p*-values are 0.04%, 0.05% and 5.22%, respectively.

[7] Corresponding *p*-values are 39.79% and 5.22%, respectively.

[8] Some values of betas in equation (11) are insignificant. Especially for mode 1 of the first sub-period the loadings are insignificant both for *HML* and *SMB*. However, for unambiguous interpretation of the market factor impact we decided to orthogonalize the market factor in each case. A similar procedure con-

$$f_t^{MO} = \alpha + e_t. \tag{12}$$

The response variable and the explanatory variables are subject to stationarity tests whose hypothesis is based on the Dickey-Fuller and the Phillips-Perron tests. Dickey-Fuller and the augmented Dickey-Fuller tests as well as Phillips-Perron tests confirm lack of unit root for each test case on 1% significance level (see: Dickey and Fuller (1979), Perron (1989), and Phillips and Perron (1988)).[9] This leads to conclusions regarding the stationarity of the analyzed variables.

We test the FF model in the two passes:

$$r_{it} - RF_t = \alpha_i + \beta_{i,HML} f_t^{HML} + \beta_{i,SMB} f_t^{SMB} + \beta_{i,MO} f_t^{MO} + e_{it}, \quad t = 1, ..., \mathrm{T}; \quad \forall i = 1, ..., 10, \tag{13}$$

$$r_{it} - RF_t = \gamma_0 + \gamma_{HML}\hat{\beta}_{i,HML} + \gamma_{SMB}\hat{\beta}_{i,SMB} + \gamma_{MO}\hat{\beta}_{i,MO} + \varepsilon_{it}, \quad i = 1, ..., 10; \quad t = 1, ..., \mathrm{T}. \tag{14}$$

Beta values are estimators of the systematic risk. The second pass estimates the beta loadings which define risk premiums. Regression parameters in (13) and (14) are estimated via GLS – following Prais-Winsten procedure and by three bootstrap methods: quantile bootstrap, BCα bootstrap, and t-bootstrap (see Efron and Tibshirani, 1993). Homoskedasticity of the residuals is confirmed using White and Breusch-Pagan methods. Therefore, the heteroscedascity correction is not required.[10]

The parameters of the second pass can be estimated by three variants:
1) the pooled time-series and cross-section estimate,
2) the „pure cross-sectional" estimate, on the basis of time series averages,
3) the Fama-MacBeth procedure that means running a cross-sectional regression at each point in time; the estimated parameters $\hat{\gamma}_0$ and $\hat{\gamma}_1$ are the average cross-sectional estimates of $\hat{\gamma}_{0t}$ and $\hat{\gamma}_{1t}$.[11] The time-series standard deviations of $\hat{\gamma}_{0t}$ and $\hat{\gamma}_{1t}$ are used to estimate the standard error of $\hat{\gamma}_0$ and $\hat{\gamma}_1$.

If the explanatory variables of regression (14) do not vary over time, and if the errors are cross-sectionally correlated but not correlated over time, then the pooled time-series and cross-sectional OLS estimate, the "pure cross-sectional" OLS estimate, and the the Fama-MacBeth procedure are identical (see Cochrane, 2001, pp. 247–250). The second pass estimates the values of beta loadings which define risk premiums. The risk premium vector is estimated using the pooled time-series and cross-section data. Independent variables (betas) remain permanent for all periods, while dependent variables constitute the returns which should by nature be random (see Cochrane 2001, p. 247). Therefore, we assume the lack of autocorrelation of the residual component. The impact of heteroskedasticity is taken into account by means of the change of variables method.[12]

cerning the orthogonalization of the market factor is applied by Fama and French (1993, p. 27–31) for the five-factor model. The loadings of all of the tested *HML*, *SMB*, *TERM* and *DEF* variables differ significantly from zero. The determination coefficient of the analyzed regression (by FF) is $R^2 = 38\%$.

[9] Phillips and Perron, and Dickey-Fuller tests are carried out for the three tested periods. 13 tested cases include the response variable for 5 portfolios formed on *BV*/*MV* and *CAP* and the 3 explanatory variables: f_t^{MO}, f_t^{HML} and f_t^{SMB}. The augmented Dickey-Fuller tests are carried out for lag, defined on the basis of minimizing the modified Akaike criterion, assuming that maximum lag equals 4. Test findings are available from the authors upon request.

[10] The covariance matrix of regression coefficients is also estimated by means of the Newey-West estimator where standard errors are corrected for autocorrelation and heteroskedasticity. The results are qualitatively similar. They are readily available upon request.

[11] $\hat{\gamma}_1$ is the vector $\hat{\gamma}_1\,[\hat{\gamma}_{HML}, \hat{\gamma}_{SMB}, \hat{\gamma}_{MO}]$.

[12] See footnote 7.

The impact of estimation errors of the true beta values in the first pass is considered by correcting the standard errors of beta loadings estimated in the second pass. With this purpose in mind Shanken's estimator is applied (see Shanken, 1992).

Table 1 presents the values of parameters of regression (13) for the full-sample and for the portfolios of mode 1 type.[13] The regression parameters estimated in "null" regressions for the first and second sub-periods are subject to Chow's stability tests. The results confirm the parameters stability in 6 out of 10 tested portfolios of mode 1 and mode 2 type.

Table 1

The parameter values of time-series regression of excess stock returns on the orthogonalized stock-market factor , f^{MO} and the Fama-French factors: f^{HML} and f^{SMB}

$$r_{it} - RF_t = \alpha_i + \beta_{i,HML} f_t^{HML} + \beta_{i,SMB} f_t^{SMB} + \beta_{i,MO} f_t^{MO} + e_{it}; \quad \forall i = 1, ..., 10$$

Mode 1: The sample period is from 1995 to 2010, T = 56 quarters								
Portfolio	Quantile bootstrap, θ^*		BCα bootstrap, θ^*		t-bootstrap	„null" regression		
i	$\hat{\theta}^*_{0.025}$	$\hat{\theta}^*_{0.975}$	$\hat{\theta}^*_{0.025}$	$\hat{\theta}^*_{0.975}$	p-value, %^{a}	$\hat{\theta}$	p-value, %^{a}	R^2 %
	$\hat{\theta}^* = \hat{\alpha}_i$					$\hat{\theta} = \hat{\alpha}_i$		
1	**–0.04**	**–0.01**	**–0.04**	**–0.01**	**1.16**	–0.02	**0.77**	88.12
5	–0.04	0.02	–0.04	0.02	32.38	–0.02	30.14	75.40
6	–0.02	0.03	–0.02	0.03	71.14	0.00	67.87	91.69
10	–0.01	0.01	–0.01	0.01	85.36	–0.00	81.35	94.89
	$\hat{\theta}^* = \hat{\beta}_{i,HML}$					$\hat{\theta} = \hat{\beta}_{i,HML}$		
1	**–0.51**	**–0.26**	**–0.50**	**–0.25**	**0.02**	–0.39	**0.00**	88.12
2	**–0.65**	**–0.37**	**–0.64**	**–0.35**	**0.02**	–0.51	**0.00**	82.62
3	**–0.41**	**–0.08**	**–0.44**	**–0.11**	**0.72**	–0.24	**0.65**	81.24
4	**0.21**	**0.66**	**0.22**	**0.67**	**0.04**	0.44	**0.04**	63.52
5	**0.40**	**0.91**	**0.45**	**1.00**	**0.00**	0.66	**0.00**	94.89
6	**–0.71**	**–0.42**	**–0.75**	**–0.44**	**0.02**	–0.56	**0.00**	91.69
10	**–0.37**	**–0.23**	**–0.36**	**–0.22**	**0.02**	–0.30	**0.00**	94.89
	$\hat{\theta}^* = \hat{\beta}_{i,MO}$					$\hat{\theta} = \hat{\beta}_{i,MO}$		
1	**0.89**	**1.10**	**0.91**	**1.13**	**0.00**	1.00	**0.00**	88.12
5	**0.77**	**1.21**	**0.79**	**1.24**	**0.00**	0.99	**0.00**	75.40
6	**0.98**	**1.24**	**0.99**	**1.24**	**0.00**	1.11	**0.00**	91.69
10	**0.92**	**1.05**	**0.92**	**1.05**	**0.00**	0.99	**0.00**	94.89
	$\hat{\theta}^* = \hat{\beta}_{i,SMB}$					$\hat{\theta} = \hat{\beta}_{i,SMB}$		
1	**0.05**	**0.32**	**0.04**	**0.31**	**1.88**	0.18	**1.50**	88.12
5	**0.37**	**0.95**	**0.39**	**0.97**	**0.00**	0.65	**0.01**	75.40
6	**1.39**	**1.74**	**1.41**	**1.76**	**0.00**	1.56	**0.00**	91.69
7	**0.90**	**1.22**	**0.87**	**1.19**	**0.00**	1.06	**0.00**	89.10
8	**0.60**	**0.97**	**0.56**	**0.95**	**0.00**	0.78	**0.00**	83.62
9	**0.45**	**0.76**	**0.46**	**0.78**	**0.00**	0.60	**0.00**	87.31
10	–0.05	0.12	–0.06	0.11	44.36	0.03	45.14	94.89

Regression parameters for all bootstrap iterations and „null" regression are estimated by GLS. Portfolio for i = 1 is formed on minimal value of BV/MV. Portfolio for i = 5 is formed on maximal value of BV/MV. Portfolio for i = 6 is formed on minimal value of CAP. Portfolio for i = 10 is formed on maximal value of CAP. $\hat{\theta}^*_{0.025}$ is the bootstrapped value of the estimator for the 2,5% level and, similarly, $\hat{\theta}^*_{0.975}$ is the bootstrapped value of the estimator for the 97,5% level. The bootstrap quantile is based on 10000 data resamples. Negative-BV stocks are excluded from the portfolios. The errors-in-variables are adjusted and follow Shanken (1992).

[a] Corresponds to the significance test for model parameters in the null hypotheses.

Bold type – the parameter is significantly different from zero at the level of 5%.

Source: own research.

[13] Parameter values for the sub-periods and for mode 2 are available on request.

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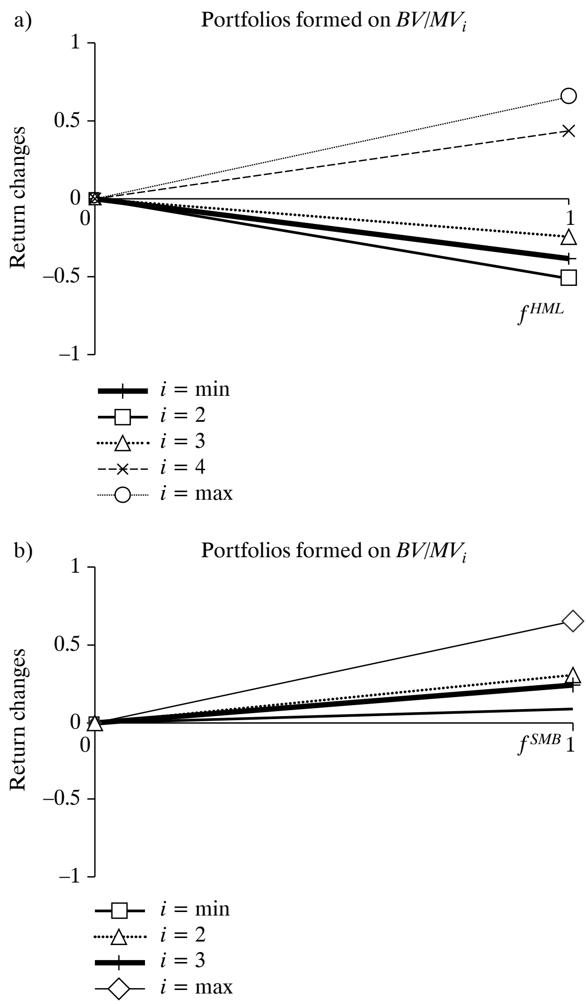
The cross-section changes of regression parameters, for the portfolios formed on mode 1 and mode 2 are similar.

For each quintile formed on *BV/MV* the *HML* regression coefficients increase from strongly negative values for the lowest quintiles to strongly positive values for the highest quintiles. *SMB* regression coefficients assume positive values for all quintiles. Calculation results show that the growth in *HML* is accompanied by the growth in returns for value stocks (high *BV/MV*) and decrease in returns for growth stocks (low *BV/MV*). The schemes of return changes on portfolios formed on *BV/MV* are presented in Fig. 1.

Figure 1

Influence of f^{HML} and f^{SMB} factors on returns of portfolios formed on BV/MV^{a}



[a] This figure shows the influence of f^{HML} (Fig. a) and f^{SMB} (Fig. b) on returns of portfolios formed on *BV/MV*. Portfolio for $i = 1$ is formed on minimal value of *BV/MV*. Portfolio for $i = 5$ is formed on maximal value of *BV/MV*. Negative-*BV* stocks are excluded from the portfolios. The sample period is from 1995 to 2010, 56 quarters.

For each quintile formed on *CAP* the *SMB* regression coefficients decrease from the smallest to biggest quintiles. Portfolios with small *CAP* give increasing returns for higher

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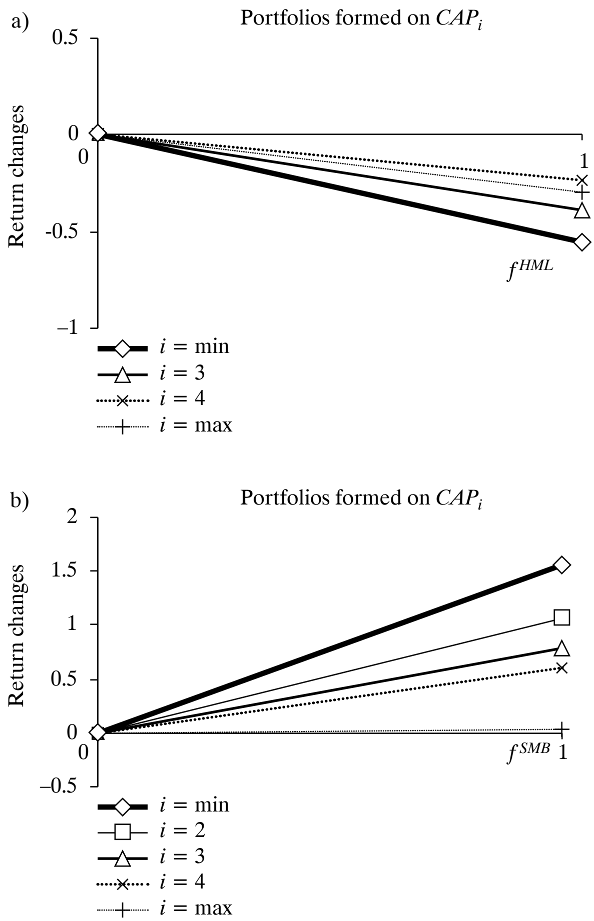
SMB. *HML* regression coefficients assume negative values for four quintiles. The schemes of return changes on portfolios formed on *CAP* are presented in Fig. 2. The R^2 coefficient estimated by „null" regression assumes values between 63.52% and 94.89%.

Figure 2

Influence of f^{HML} and f^{SMB} factors on returns of portfolios formed on capitalization, *CAP* [a]



[a] This figure shows the influence of of f^{HML} (Fig. a) and f^{SMB} (Fig. b) on returns of portfolios formed on capitalization, *CAP*. Portfolio for $i=1$ is formed on minimal value of *CAP*. Portfolio for $i = 5$ is formed on maximal value of *CAP*. Negative-*BV* stocks are excluded from the portfolios. The sample period is from 1995 to 2010, 56 Quarters.

Table 2 presents the values of parameters of regression (14). The risk premiums for the portfolios formed on mode 1 and mode 2 are similar.

The loadings on betas estimated in "null" regressions are insignificantly different from zero for all the tested periods. The corresponding *p*-values are higher than 21%. However, the risk premiums γ_{HML} and γ_{SMB}, estimated with quantile bootstrap and BCα bootstrap, take positive values for the second sub-period and for the whole sample at the level of 5%. In the first sub-period γ_{HML} estimated via quantile bootstrap and *t*-bootstrap takes positive values at the level of 10%.

Table 2

The risk premium vector (γ) values estimated from the second-pass regression for the Fama-French model

$$r_{it} - RF_t = \gamma_0 + \gamma_{HML}\hat{\beta}_{i,\,HML} + \gamma_{SMB}\hat{\beta}_{i,\,SMB} + \gamma_{MO}\hat{\beta}_{i,\,MO} + \varepsilon_{it};\quad i = 1, ..., 10; t = 1, ..., T$$

Mode		Quantile bootstrap, θ^*		BCα bootstrap, θ^*		t-bootstrap	"null" regression	
	Parameter	$\hat{\theta}^*_{2.5\%}$	$\hat{\theta}^*_{97.5\%}$	$\hat{\theta}^*_{2.5\%}$	$\hat{\theta}^*_{97.5\%}$	p-value, % [a]	$\hat{\theta}$	p-value, % [a]
The sample period is from 1995 to 2010, T = 56 quarters								
1	$\hat{\gamma}_0$	–0.14	0.07	–0.20	0.02	14.50	–0.05	67.03
	$\hat{\gamma}_{HML}$	**0.00**	**0.05**	**0.01**	**0.06**	**0.48**	0.03	21.51
	$\hat{\gamma}_{MO}$	–0.08	0.13	–0.03	0.21	18.80	0.05	71.05
	$\hat{\gamma}_{SMB}$	**0.00**	**0.03**	*–0.01*	*0.02*	21.88	0.01	56.47
2	$\hat{\gamma}_0$	–0.15	0.08	–0.19	0.05	29.44	–0.04	70.65
	$\hat{\gamma}_{HML}$	**0.00**	**0.05**	**0.01**	**0.06**	**0.78**	0.03	23.83
	$\hat{\gamma}_{MO}$	–0.09	0.14	–0.05	0.20	37.92	0.04	76.60
	$\hat{\gamma}_{SMB}$	**0.00**	**0.03**	**0.00**	**0.03**	10.30	0.01	50.07
The sample period is from 1995 to 2005, T = 36 quarters								
1	$\hat{\gamma}_0$	–0.08	0.06	–0.07	0.09	69.62	–0.01	85.98
	$\hat{\gamma}_{HML}$	*–0.01*	*0.05*	**0.00**	**0.05**	*10.00*	0.02	44.57
	$\hat{\gamma}_{MO}$	–0.08	0.06	–0.11	0.05	92.84	–0.00	95.97
	$\hat{\gamma}_{SMB}$	–0.01	0.02	–0.01	0.02	59.78	0.00	84.04
The sample period is from 2005 to 2010, T = 20 quarters								
1	$\hat{\gamma}_0$	–0.10	0.08	–0.11	0.07	75.52	–0.01	92.49
	$\hat{\gamma}_{HML}$	**0.00**	**0.06**	**0.01**	**0.07**	**0.06**	0.04	30.10
	$\hat{\gamma}_{MO}$	–0.07	0.11	–0.06	0.12	55.06	0.02	85.76
	$\hat{\gamma}_{SMB}$	**0.01**	**0.04**	**0.01**	**0.04**	**0.50**	0.02	40.97
2	$\hat{\gamma}_0$	– 0.19	0.02	**–0.32**	**–0.04**	**0.02**	–0.11	46.53
	$\hat{\gamma}_{HML}$	**0.00**	**0.06**	**0.01**	**0.09**	**0.40**	0.03	37.27
	$\hat{\gamma}_{MO}$	*–0.02*	*0.20*	**0.04**	**0.33**	**0.00**	0.11	48.06
	$\hat{\gamma}_{SMB}$	**0.00**	**0.04**	**0.00**	**0.04**	*7.68*	0.02	52.88

Regression parameters for all bootstrap iterations and "null" regression are estimated by GLS. Portfolio for $i = 1 – 5$ are formed on BV/MV. Portfolios for $i = 6 – 10$ are formed on capitalization, CAP. $\hat{\theta}^*_{2.5\%}$ is the bootstrapped value of the estimator for the 2,5% level and, similarly, $\hat{\theta}^*_{97.5\%}$ is the bootstrapped value of the estimator for the 97,5% level. The bootstrap quantile is based on 10000 data resamples. In mode 1 negative-BV stocks are excluded from the portfolios. In mode 2 speculative stocks are excluded from the portfolios. It is assumed that speculative stocks meet one of the following two conditions: 1) $MV/BV>100$ and $rit > 0$, 2) $ROE<0$ and $MV/BV>30$ and $rit > 0$, where MV is the stock market value, ROE is the return on book value (BV), rit is the return of portfolio i in period t.
[a] Corresponds to the significance test for model parameters in the null hypotheses.
Bold type – the parameter is significantly different from zero at the level of 5%. *Italic type* – the parameter is significantly different from zero at the level of 10%.

Source: own research.

Less clear results are obtained for the risk premium vectors γ_{MO}. Using the whole sample data, only the BCα bootstrap is positive at the level of 10% for mode 1 type portfolios. This confirms earlier findings obtained for the US market, that the factor f^{MO} does not appear to be important in the ICAPM model (see, for example, Petkova, 2006; Fama and French, 1992; Jagannathan and Wang, 1996 or Lettau and Ludvigson, 2001).

In the whole tested period γ_{HML} does not change and equals approx. 3% per quarter. Component γ_{SMB} is lower and equals about 1–2%. Investors on the Polish market reveal a higher and positive risk premium in the case of different *BV*/*MV* stock parameters.

The results of estimation of FF model parameters, on the basis of classic asymptotical methods, do not support Conjecture 1. However, after using bootstrap method the results are in line with Conjecture 2.

ME is tested under the assumption that errors of the regression (13) are iid. Also, we test the normality of residuals.[14] We employ three efficiency tests, the GRS test, the asymptotic Wald test and bootstrap tests. The empirical results are reported in Table 3.

Under iid assumption, the GRS and asymptotic Wald tests reject ME of the FF portfolios for the first sub-period and for the whole sample for portfolios formed under mode 2 assumption at the 5% significance level.

However, the bootstrapped Wald test, W^*, does not reject efficiency for investigated periods. We may conclude that the FF model generates ME portfolios on the WSE when stock returns are assumed to come from iid models, which is consistent with Conjecture 1.

The validity of Conjectures 1 and 2 confirms Conjecture 3.

Table 3

The results of multifactor-efficiency tests

	Quantile bootstrap, W*			W		GRS-*F*	
	$\hat{W}^*_{5\%}$	$\hat{W}^*_{10\%}$	*p*-value (χ^2), %	Statistics value	*p*-value (χ^2), %	Statistics value	*p*-value (*F*), %
Panel A: Present research							
	$r_{it} - RF_t = \alpha_i + \beta_{i,HML} f_t^{HML} + \beta_{i,SMB} f_t^{SMB} + \beta_{i,MO} f_t^{MO} + e_{it}; \forall i = 1$						
Mode	Period: 1995–2010						
1	70.73	61.50	97.88	13.35	20.47	1.10	38.10
2	84.06	73.55	90.52	26.46	0.32	2.19	3.73
Period: 1995–2005							
1	102.91	87.41	58.61	45.01	0.00	3.23	0.96
Period: 2005–2010							
1	131.18	100.17	97.82	9.83	45.54	0.43	89.05
2	172.94	134.00	97.83	17.96	5.57	0.79	64.78
Panel B: Chou and Zhou (2006), Fama-French's factors							
Period: 1964–1993			0.03		<0.01		0.01
Panel C: Chou and Zhou (2006), CRSP index							
Period: 1926–1995			6.80		0.57		3.30
Period: 1986–1995			38.00		21.04		28.43

$H_0 = \alpha_i = 0; \forall i = 1, \ldots, N$. W is the Wald statistics. GRS-*F* is the *F*-statistics of Gibbons et al. (1989). In mode 1 negative-*BV* stocks are excluded from the portfolios. In mode 2 speculative stocks are excluded from the portfolios. In panel B the authors examine the joint efficiency of the Fama-French's factors in: $r_{it} - RF_t = \alpha_i + \beta_{i,HML} f_t^{HML} + \beta_{i,SMB} f_t^{SMB} + \beta_{i,MO}(RM_t - RF_t) + e_{it}$, where r_{it}'s are monthly returns on 25 Fama-French's portfolios and $RMt - RFt$ is the excess return on a market index. In panel C the authors examine the efficiency of the CRSP value-weighted index in the standard market model: $R_t = \alpha + \beta r_{pt} + e_t$, where R_t is a vector of returns on 10 CRSP size decile portfolios in excess of the 30-day T-bill rate. The bootstrap quantile is based on 10000 data resamples.

Source: own research.

[14] The Shapiro-Wilk tests confirm the residuals normality for the whole sample in 6 out of 10 tested portfolios.

Also, we compare ME procedure results to other studies on American market (see Chou and Zhou, 2006). The chosen results are specified in panel B and panel C of Table 3. In the case of FF model the p-values from the GLS, W and W* tests suggest a strong rejection. However the bootstrap tests have greater p-values than the non-bootstrap ones.

4. Conclusions

The use of bootstrap to test the Fama-French application is presented for WSE stocks.

In Tables 1, 2 and 3, a detailed comparison in made between the quantile and BCα bootstrap confidence intervals on one side and classical asymptotic confidence intervals on the other side. It is clear that critical parameters, like γ_{HML}, γ_{SMB} and W, have significantly different confidence intervals; therefore, these generate different results in significance tests. For example, in Table 2 the confidence interval for γ_{HML} based on BCα bootstrap generates a confidence interval not including zero. This means that according to bootstrap technique the unknown parameter γ_{HML} is significant. A corresponding classical test, based on asymptotic normal distribution, generates the p-value much bigger than 10% which means that according to classical theory the parameter would not be significant.

According to well-established facts, regarding the sample of moderate size (see Efron and Tibshirani, 1993), the bootstrap technique provides better and more reliable results. Bootstrap technique, due to its nonparametric nature, better follows the true sampling distribution of the estimator. Therefore, in the FF model, bootstrap technique provides consistently higher quality results.

The conducted research, referring to the main objective of the paper, leads to the following conclusions:

1. The bootstrapped Wald test does not reject ME for the tested FF portfolios. However, the GRS and asymptotic Wald tests reject ME for most tested cases.
2. Systematic risk components of FF portfolios, $\hat{\beta}_{i,\,HML}$ and $\hat{\beta}_{i,\,SMB}$, estimated by bootstrap, are priced.
3. The valuation of FF portfolios, formed on WSE, is consistent with the pricing that could be observed in the conditions of ICAPM validity.

The study concerning an additional aim of the paper leads to the further conclusions:

4. Long investments in companies with high *BV*/*MV* show higher returns for growing *HML* and *SMB* values.
5. Long investments in companies with small capitalization show higher returns for growing *SMB* and decreasing *HML* values.
6. In the whole period risk prices γ_{HML} and γ_{SMB} do not change and equal approx. 3% and 1–2% per quarter.
7. Speculative stocks (defined in Section 3) do not affect the values of systematic risk and risk premium components.

Received: 11 January 2014

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WIELOCZYNNIKOWA EFEKTYWNOŚĆ PORTFELI FAMY-FRENCHA FORMOWANYCH NA GPW W WARSZAWIE: ZASTOSOWANIE METOD BOOTSTRAP

Streszczenie

Artykuł przedstawia bootstrapową ocenę wieloczynnikowej efektywności portfeli Famy-Frencha formowanych na polskim rynku akcji. Zastosowane metody oceniają charakter zmian stóp zwrotu w zależności od zmian czynników Famy-Frencha. Wektory ryzyka i premii za ryzyko osza-

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cowano w okresie 1995–2010 oraz dwóch podokresach. Zastosowanie modelu Famy-Frencha do budowy portfeli inwestycyjnych pozwala na wysunięcie wielu wskazówek użytecznych dla inwestorów i zarządzających portfelami akcji.

Wyniki analizy pokazują, że zastosowanie metod bootstrap pozwala na dokładniejszą estymację badanych parametrów. Umożliwia to lepszą ocenę zmian stóp zwrotu niż klasyczne procedury oparte na założeniach rozkładów normalnych.

Słowa kluczowe: model Famy-Frencha, metoda bootstrap, zmiany stóp zwrotu, ryzyko systematyczne

MULTIFACTOR-EFFICIENCY OF THE FAMA-FRENCH PORTFOLIOS FORMED ON THE WARSAW STOCK EXCHANGE: BOOTSTRAP METHOD APPLICATION

Abstract

This paper presents the use of bootstrap method to assess the multifactor-efficiency of Fama-French portfolios formed on the Warsaw Stock Exchange. The presented methods estimate the nature of return changes influenced by the Fama-French factors. The risk and risk premium vectors are determined for full-sample observations (1995–2010) and two sub-periods.

Using of the Fama-French model to forming the investment portfolios leads to a number of conclusions that may be useful for investors and portfolio managers. The results ofthe analysis show that application of bootstrap methods allows a better estimation of the parameters concerned. This provides a better approximation of returns changes than the classic procedures based on the assumption of normal distribution.

Key words: Fama-French model, bootstrap method, return changes, systematic risk

JEL classification: G11, G12.

МНОГОФАКТОРНАЯ ЭФФЕКТИВНОСТЬ ПОРТФЕЛЯ ФАМЫ-ФРЕНЧА НА БИРЖЕ ЦЕННЫХ БУМАГ В ВАРШАВЕ: ПРИМЕНЕНИЕ МЕТОДА БУТСТРЕП

Резюме

Статья представляет бутстрепную оценку многофакторной эффективности портфеля Фамы-Френча на польском рынке акций. Примененные методы оценивают характер изменений норм окупаемости в зависимости от изменений факторов Фамы-Френча. Векторы риска и премии за риск были оценены за период 1995-2010, а также в двух субпериодах. Применение модели Фамы-Френча для построения инвестиционных портфелей, позволяет сформулировать указания, полезные для инвесторов и управляющих портфелями акций. Результаты анализа указывают, что применение метода бутстреп позволяет сделать более точную эстимацию изучаемых параметров, что помогает лучше оценить изменения норм окупаемости по сравнению с классическими процедурами, опирающимися на нормальное распределение.

Ключевые слова: модель Фамы-Френча, метод бутстреп, изменения норм окупаемости, систематический риск